DOMAIN INTEGRAL EQUATIONS FOR ELECTROMAGNETIC BAND-GAP SLAB SIMULATIONS

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Abstract: Electromagnetic band-gap substrates offer advantages regarding cross-coupling in sub-mm range imaging array applications. Electromagnetic scattering by such substrates may effectively be formulated in terms of integro-differential equations. The scattered fields are evaluated via the spectral domain in which the 3-D problem reduces to an infinite system of coupled 1-D integro-differential equations. The associated matrix-vector products are dominated by FFTs. For moderate frequencies an elementary preconditioner in combination with a prudent initial estimate usually suffices to reach rapid convergence using the transpose-free quasi-minimal residual method.

INTRODUCTION

Electromagnetic phenomena in the sub-mm range are of growing interest in the areas of astronomy and atmospheric research. An imaging array of antennas would comprise a useful device in disclosing this frequency range. Such an imaging array has to be mounted on top of a substrate. In order to reduce cross-coupling and to suppress surface waves that would be present in homogeneous dielectric substrates, electromagnetic bandgap (EBG) structures, e.g. woodpile structures (cf. Figure 1), provide viable alternatives to dielectric substrates, albeit that for the mechanical support of the antenna elements an interleaving membrane is still required, and this influences the electromagnetic behaviour of the resulting structure (cf. Reynolds *et al.* [1]). Gonzalo *et al.* [2] are presently developing a prototype imaging array mounted on a woodpile operating about 500 GHz. In order to assess alternative designs for antennas on EBG substrates, flexible, accurate, and reliable numerical modelling methods are required. A genuine EBG structure is periodic in every direction. Its band-diagram can be analysed with the aid of the plane-wave method. A practical EBG structure is finite in one or more directions. Hence, a basis of homogeneous plane waves ceases to be efficient, and the computational complexity increases considerably. Bell *et al.* [3] used a transfer-matrix method (TMM) to analyse slabs of EBG materials. To circumvent the intrinsic instabilities in the TMM, Gralak *et al.* [4] have employed scattering matrices or, equivalently impedance matrices.

We are interested in the design of finite arrays of patch antennas on an EBG substrate of finite thickness. The non-periodicity adds to the computational complexity. Central to this problem is the calculation of the Green's function associated with a single current cell. As regards woodpiles (for which subsequent layers retain invariance in alternating transverse directions (cf. Figure 1)) a method based on scattering (or impedance) matrices is probably among the most efficient and accurate methods. Once more general geometries are considered, the domain integral equation for the electric field described in this paper may be expected to yield a competing performance. Initially, we assume phased periodic sources in the transverse directions so as to exploit the pertaining periodicity, resulting in an infinite system of coupled 1-D integral equations. In the vertical direction, we expand the field in terms of piecewise linear splines. The complexity of the associated efficient matrix-vector product is dominated by FFTs. The resulting linear system may be solved with the aid of an iterative technique for a sequence of consecutive phases, in order to go from phased periodic sources to localised sources via array summation (ASM, cf. Munk *et al.* [5]), or for a sequence of frequencies, in order to obtain the reflection/transmission properties of the EBG slab inside the band-gap and beyond. Despite the large number of unknowns, storage requirements are limited, while the computation time taken by the iterative solver is kept in check through the use of an appropriate preconditioner and initial estimates based on results generated earlier in the sequence. Once the Green's functions for all possible source and receiver cell combinations have been stored, one can search for the optimum antenna design with the aid of a discrete optimisation algorithm, involving an electric-field integral equation at every stage.

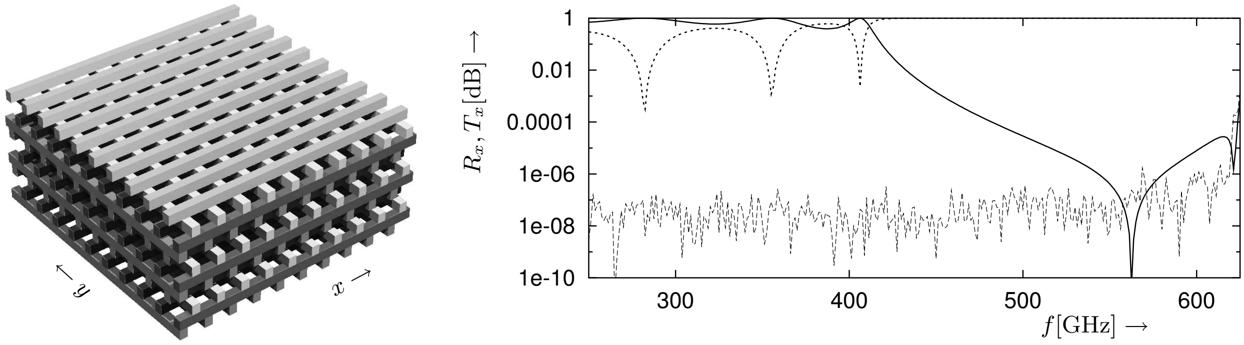

Figure 1: Twelve layers of a woodpile (left) with bars of dimensions $71 \times 71\mu$m, periods $(a, a, 1.22a)$ with $a = 232.9\mu$m, and relative permittivity $\varepsilon_\mathrm{r} = 11.7$. The respective solid, dotted and dashed lines (right) represent the transmitted and reflected power, and the power mismatch, due to a y-polarised unit-amplitude plane wave from above at normal incidence.

FORMULATION OF THE PROBLEM

We consider a slab consisting of a material with medium properties that are periodic the x–y plane (transverse plane) and vary with respect to z for $-d < z < 0$. Outside the slab the material is homogeneous. The spatial derivatives of the dielectric contrast are assumed bounded throughout. The permeability is taken constant. We consider a periodic array of current sources with linearly progressing phase, where the periodicity of the sources coincides with the periodicity of the material variation in the slab. This results in Bloch modes in the directions of periodicity. The original problem for a single source can subsequently be solved using the ASM. Regarding the periodic situation, the Bravais lattice in the transverse plane has basis vectors $\mathbf{a}_1$ and $\mathbf{a}_2$, that span the cell $S_\mathrm{t}^\mathrm{cell}$, and reciprocal basis vectors $\mathbf{b}_1$ and $\mathbf{b}_2$, such that $(\mathbf{a}_i, \mathbf{b}_j) = 2\pi\delta_{i,j}$. The spectral constituents in the transverse plane are written as $F^\mathbf{m}(\mathbf{x}_\mathrm{t}, \boldsymbol{\varphi}) = e^{i\mathbf{k}_\mathrm{t}\cdot\mathbf{x}_\mathrm{t}}$ with $\mathbf{k}_\mathrm{t} = \mathbf{k}_\mathrm{t}^\mathbf{m} + \mathbf{k}_\mathrm{t}^{\boldsymbol{\varphi}}$, in which $\mathbf{k}_\mathrm{t}^\mathbf{m} = m_1\mathbf{b}_1 + m_2\mathbf{b}_2$ with $m_1, m_2 \in \mathbb{Z}$, and $\mathbf{k}_\mathrm{t}^{\boldsymbol{\varphi}} = (\varphi_1\mathbf{b}_1 + \varphi_2\mathbf{b}_2)/(2\pi)$, with $\varphi_1, \varphi_2 \in [-\pi, \pi)$ respectively take into account the periodicity and the linearly progressing phase, while $\mathbf{x}_\mathrm{t} = x\mathbf{u}_x + y\mathbf{u}_y$. The fields are considered to be time-harmonic with an $e^{-i\omega t}$ time dependence. We separate the field in an incident field $\{\mathbf{E}^i, \mathbf{H}^i\}$ corresponding to $\mathbf{J}^i$ in the homogeneous background medium, with $\varepsilon = \varepsilon_1$ and $k_1 = \omega\sqrt{\varepsilon_1\mu_0}$, and a scattered field $\{\mathbf{E}^s, \mathbf{H}^s\}$ due to the periodic structure in the slab. For the scattered field we introduce a vector potential $\mathbf{A}$, via $\mathbf{H}^s = -i\omega\varepsilon_1\nabla\times\mathbf{A}$ and apply the Lorenz gauge. The spectral field quantities, such as the potential, and the spectral counterpart of the dielectric contrast, $\chi(\mathbf{x}) = (\varepsilon(\mathbf{x})/\varepsilon_1 - 1)$, are related to their spatial counterparts via

$$\mathbf{a}(\mathbf{m}, z, \boldsymbol{\varphi}) = \int_{S_\mathrm{t}^\mathrm{cell}} e^{-i\mathbf{k}_\mathrm{t}\cdot\mathbf{x}}\mathbf{A}(\mathbf{x})\,\mathrm{d}A \qquad \text{and} \qquad \tilde{\chi}(\mathbf{m}, z) = \int_{S_\mathrm{t}^\mathrm{cell}} e^{-i\mathbf{k}_\mathrm{t}^\mathbf{m}\cdot\mathbf{x}}\chi(\mathbf{x})\,\mathrm{d}A. \tag{1}$$

The spectral contrast current, normalised to $-i\omega\varepsilon_1$, and vector potential respectively satisfy

$$\mathbf{j}(\mathbf{m}, z, \boldsymbol{\varphi}) = \sum_{\mathbf{m}'=-\infty}^{\infty} \tilde{\chi}(\mathbf{m}-\mathbf{m}', z)\mathbf{e}(\mathbf{m}', z, \boldsymbol{\varphi}) \;\text{ and }\; \mathbf{a}(\mathbf{m}, z, \boldsymbol{\varphi}) = \int_{-d}^{0} g(\mathbf{m}, z - z', \boldsymbol{\varphi})\mathbf{j}(\mathbf{m}, z', \boldsymbol{\varphi})\,\mathrm{d}z', \tag{2}$$

where $g(\mathbf{m}, z, \boldsymbol{\varphi}) = e^{ik_z|z|}/(-2ik_z)$ denotes the 1-D Green's function, and $k_z = \sqrt{k_1^2 - \|\mathbf{k}_\mathrm{t}\|_2^2}$, with $\mathrm{Im}(k_z) > 0$ or $k_z \geq 0$ is the vertical wavenumber. Together with Eq. (2),

$$\mathbf{e}^i(\mathbf{m}, z, \boldsymbol{\varphi}) = \mathbf{e}(\mathbf{m}, z, \boldsymbol{\varphi}) - k_1^2\mathbf{a}(\mathbf{m}, z, \boldsymbol{\varphi}) - [i\mathbf{k}_\mathrm{t} + \mathbf{u}_z\partial_z]\left\{[i\mathbf{k}_\mathrm{t} + \mathbf{u}_z\partial_z]\cdot\mathbf{a}(\mathbf{m}, z, \boldsymbol{\varphi})\right\} \tag{3}$$

comprises an infinite system of coupled integro-differential equations for the spectral electric field, $\mathbf{e} = \mathbf{e}(\mathbf{m}, z, \boldsymbol{\varphi})$. For an antenna problem, the incident field, $\mathbf{e}^i(\mathbf{m}, z, \boldsymbol{\varphi})$, is generated by a transverse current sheet distribution located at $z = 0$. The spectral current distributions are available in closed form (cf. Van Beurden *et al.* [6]). Here we assume that a plane wave is incident from the half space $z > 0$. Note that for $z \geq 0$ and $z \leq -d$, Eq. (3) is an integral representation for the electric field.

DISCRETISATION

We expand $\mathbf{j}(\mathbf{m}, z, \boldsymbol{\varphi})$ and $\mathbf{e}(\mathbf{m}, z, \boldsymbol{\varphi})$ in terms of equidistant linear splines in the z-direction. We take a finite set of spectral constituents in the transverse plane, and truncate the summation in Eq. (2). We discretise the the scattered electric field via the Galerkin procedure, and use a $\tilde{}$ to denote discretised quantities. The resulting system of linear equations reads $\tilde{\mathbf{e}}[\mathbf{m}, n, \boldsymbol{\varphi}] - \tilde{\mathbf{e}}^s[\mathbf{m}, n, \boldsymbol{\varphi}] = \tilde{\mathbf{e}}^i[\mathbf{m}, n, \boldsymbol{\varphi}]$, with $\tilde{\mathbf{e}}^s[\mathbf{m}, n, \boldsymbol{\varphi}] = \sum_{\mathbf{m}', n'} \mathsf{D}[\mathbf{m}, n - n', \boldsymbol{\varphi}] \mathsf{M}[\mathbf{m} - \mathbf{m}', n'] \tilde{\mathbf{e}}[\mathbf{m}', n', \boldsymbol{\varphi}]$, which can be efficiently computed using FFTs. Note that we do not take the weak form of the electric field, i.e., $\tilde{\mathbf{e}}$ follows from $\mathbf{e}$ using point matching. Owing to this lumping procedure, which is accurate to second order, we do not need additional boundary conditions, since the contrast currents and hence the scattered field vanish outside the EBG slab. The matrices D and M represent the discretised counterparts of the differential operator and the material properties in Eqs. (3) and (2), respectively. For differentiable permittivity functions, convergence of our numerical scheme can be demonstrated. The system of equations can efficiently be solved using an iterative technique. For the numerical example in Figure 1 — a frequency sweep of the reflected and transmitted power as a result of a plane-wave incident on a twelve-layer woodpile from above at normal incidence and polarised along the y-direction — we have solved the sweep of linear systems of 300,000 unknowns via the transpose-free quasi-minimal residual (TFQMR) method (cf. Freund [7]), using a simple regularised diagonal preconditioner. At the high-frequency end of the band-gap convergence is difficult to attain. We expect that this can be remedied through the use of a more sophisticated, physics based, preconditioner.

CONCLUSIONS

We have proposed an efficient numerical scheme for electromagnetic scattering by a slab of electromagnetic bandgap material. This scheme is the engine for the construction of the Green's functions for an non-periodic source on top of the slab.

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